Optimum Placement of Controls for Static Deformations of Space Structures

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Many large space structures, such as large antennas, have to maintain a fairly exact shape to operate satisfactorily. Such structures require active and passive controls to maintain their accurate shape under disturbances. The present paper is concerned with optimum placement of controls for correcting static deformations. Both force actuators and heaters are considered for controls. A formulation of design against the worst disturbance is derived. A beam example is employed to demonstrate the procedure.

Introduction

In the design of large space antennas, one of the most stringent design requirements is that of surface accuracy.[1,2] While studies have shown that in some cases high-surface accuracies may be maintained with passive methods,[3] it is expected that for many applications active controls may be needed. The disturbances which affect the shape of space structures may be divided into two types. One type is transient, which leaves the structure unchanged once damped out. Such disturbances usually call for active or passive controls which enhance the damping of the structure. The second type of disturbance is typified by fixed deformations (e.g., due to manufacturing errors[4]) or those which are slowly varying and may be considered quasisteady. These later disturbances may be offset by slowly applied, long-acting corrections. Most research to date has concentrated on the first type of disturbance and the use of damping actuators.[5] There has been less research on countering quasisteady disturbances. Most of the work on active control of quasisteady disturbances is related to active control of optical systems such as mirrors (see Ref. 6 for a survey of the state of the art in 1978). Most of the actuators employed are force actuators (e.g., Refs. 7-11) which apply forces to the space structures in order to control its shape. Bushnell[7] characterizes some such actuators (e.g., Refs. 12 and 13) as displacement actuators because they are stiff enough to enforce a given displacement at a point. Another variation of the force actuator is one which effects a change in the length of a member by reeling a cable in or out by a screw mechanism in a truss structure. This type of approach is used on some antennas (e.g., Ref. 14) to correct fabrication errors, albeit on the ground rather than in orbit. A recently proposed alternative[15] is the use of applied heating and cooling to parts of the structure. The present paper is concerned with the optimal placement of such force or temperature controls in the structure.

One of the major problems in selecting the locations of the actuators is the choice of disturbances that have to be designed against. The problem is difficult because the exact disturbances cannot be expected to be known in advance. Furthermore, it is easy to show that for any set of actuator locations it is possible to find a disturbance which cannot be controlled by these actuators. One therefore cannot design for the worst possible disturbance. In the present work it is assumed that the actuators are designed to control disturbances which do not vary fast in space. That is, only disturbances having large enough "wavelengths" are considered. The justification for such an approach is that disturbances with small wavelengths are not likely to have large amplitudes. The limitation on the wavelength of the disturbance is enforced by assuming it to be a linear combination of a convenient set of functions (such as the first few vibration modes).

The paper starts with a review of the governing equations for the optimal control forces or temperatures based on Ref. 15. The formulation of the problem of finding the linear combination of a set of given functions which is the worst disturbance is derived. Finally, an optimization procedure which is used to find the optimum locations of the controls is described. A beam example is used to demonstrate the procedure.

Shape Control of Unrestrained Structures

Applied Heating Control

The structure is assumed to be in space and possess rigid body degrees of freedom. The structure is defined over some region Ω in space and it is assumed that its desired shape has been distorted by an amount which is described by a displacement vector $\psi(Q)$ where Q is a point in Ω. It is also assumed that the distortion does not have any rigid body motion component, that is,

$$\int_{\Omega} \rho\psi \cdot R_i \mathrm{d}\Omega = 0 \qquad i=1,...,6 \tag{1}$$

where R_i are the rigid body modes, ρ is the density of the structure per unit volume, and a dot denotes a scalar product. If Eq. (1) is violated, the correction is a matter for position and orientation control rather than shape control.

The distortion of the structure is now minimized by applying heating to selected locations on the structure. It is assumed that elements with very large coefficients of expansion (α) relative to the rest of the structure are installed at these locations. Thus, the heated high α elements are very small compared to the rest of the structure and can be considered to be concentrated at discrete points.

To solve for the deformation of a free structure in space subject to point heating it is convenient to start by calculating the deformations when the structure has been arbitrarily restrained against rigid body motion. The solution can be obtained readily by an analytical or numerical approach. The displacement field u_0 obtained this way is then purged of rigid body motion; that is, the final displacement field u is given as

$$u = u_0 + \sum_{i=1}^{6} \alpha_i R_i \tag{2}$$

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where the α_i are obtained from the equations

$$\int_\Omega \rho u \cdot R_i \mathrm{d}\Omega = 0 \qquad i=1,\ldots,6 \tag{3}$$

Any linear combination of rigid body modes is also a rigid body mode. Therefore, one can require that the modes R_i be normalized so that

$$\int_\Omega \rho R_i \cdot R_j \mathrm{d}\Omega = m\delta_{ij} \tag{4}$$

where δ_{ij} is the Kronecker delta and m the mass of the structure. With this normalization Eqs. (2) and (3) yield

$$\alpha_i = -\frac{1}{m}\int_\Omega \rho u_0 \cdot R_i \mathrm{d}\Omega \tag{5}$$

Assume that the structure is controlled by applying temperature changes ΔT_i, $i=1,\ldots,n$ at n control points. Assume that the displacement field per unit change in termperature at the ith control point is u_i

$$u_i = u_{0i} + \sum_{k=1}^{6} \alpha_{ik} R_k \tag{6}$$

where u_{0i} is the solution with convenient boundary conditions and α_{ij} are obtained from u_{0i} via Eq. (5). The total displacement due to the original disturbance and the controls is

$$u_T = \psi + \sum_{i=1}^{n} u_i \Delta T_i \tag{7}$$

The best values of ΔT_i are those that will most effectively nullify ψ, that is, cause U_T to be close to zero. A common measure of the smallness of u_T is based on the rms value

$$u_{\mathrm{rms}}^2 = \frac{1}{v_0}\int_\Omega u_T^2 \mathrm{d}\Omega \tag{8}$$

where v_0 is a reference volume and the square of a vector denotes its scalar product with itself. The necessary condition for a minimum is

$$\frac{\partial U_{\mathrm{rms}}}{\partial \Delta T_j} = \frac{2}{v_0}\int_\Omega \left(\psi + \sum_{i=1}^{n} u_i \Delta T_i\right) \cdot u_j \mathrm{d}\Omega = 0 \quad j=1,\ldots,n \tag{9}$$

Equation (9) is a system of n linear algebraic equations which may be written as

$$A\Delta T = r \tag{10}$$

where the component a_{ij} of the matrix A is

$$a_{ij} = \frac{1}{v_0}\int_\Omega u_i \cdot u_j \mathrm{d}\Omega \tag{11}$$

and the jth component of the right-hand side, r_j, is

$$r_j = -\frac{1}{v_0}\int_\Omega u_i \cdot u_j \mathrm{d}\Omega \tag{12}$$

with ΔT a vector of control temperatures. The integrals required for Eqs. (11) and (12) can be quite complex even for simple problems. When the density ρ is uniform, a_{ij} and r_j may be expressed in terms of simpler integrals. Substituting from Eq. (6) to Eq. (11) obtains

$$a_{ij} = \int_\Omega \left(u_{0i} + \sum_{k=1}^{6} \alpha_{ik} R_k\right) \cdot \left(u_{0j} + \sum_{\ell=1}^{6} \alpha_{j\ell} R_\ell\right) \mathrm{d}\Omega$$

$$= \int_\Omega u_{0i} \cdot u_{0j} \mathrm{d}\Omega - \frac{m}{\rho}\sum_{k=1}^{6} \alpha_{ik}\alpha_{jk} \tag{13}$$

Similarly, using Eqs. (1), (6), and (12) yields

$$r_j = -\int_\Omega \psi \cdot u_{0j} \mathrm{d}\Omega \tag{14}$$

The ratio of controlled to uncontrolled rms distortion g is given by

$$g^2 = \int_\Omega u_T^2 \mathrm{d}\Omega \Big/ \int_\Omega \psi^2 \mathrm{d}\Omega \tag{15}$$

It is easy to check that

$$g^2 = \frac{r_{m0}^2 - 2r^T\Delta T + \Delta T^T A \Delta T}{r_{m0}^2} = 1 - \frac{r^T \Delta T}{r_{m0}^2} \tag{16}$$

where r_{m0} represents the rms distortion associated with ψ.

Force Control

The analysis for control by applied concentrated force is more complicated because the applied forces have to be self-equilibrating. As before it is assumed that there are n control points. The displacement shape u_{0i} due to a unit control force at point i is solved by selecting a convenient set of supports. Then, the corresponding set of rigid body mode contribution is added as in the thermal control case. The analysis is identical with one exception: the applied set of forces has to be in equilibrium. This is written as

$$\bar{R}_i^T F = 0 \qquad i=1,\ldots,6 \tag{17}$$

where F is the vector of control forces and $\bar{R}_i$ a vector of the components of the ith rigid body motion of the n control points.

Defining a matrix R composed of the column vectors $\bar{R}_i$, Eq. (17) is written as

$$R^T F = 0 \tag{18}$$

It is possible to take care of Eq. (18) by employing Lagrange multipliers. However, it is often possible to identify a nonsingular submatrix R_B of R so that some of the components of the force vector can be expressed in terms of the others. The partitioning may be written as

$$[R_A R_B]\begin{Bmatrix} F_A \\ F_B \end{Bmatrix} = 0 \tag{19}$$

where F_A and F_B are the corresponding parts of F. Then,

$$F = \begin{bmatrix} I \\ -R_B^{-1} R_A \end{bmatrix} F_A \equiv S F_A \tag{20}$$

The rms distortion is given now by

$$u_{\mathrm{rms}}^2 = (1/v_0)\int_\Omega \left(\psi + \sum_{i=1}^{n} u_i F_i\right)^2 \mathrm{d}\Omega \tag{21}$$

or in matrix terms

$$u_{\mathrm{rms}}^2 = r_{m0}^2 - 2r^T F + F^T A F \tag{22}$$

substituting from Eq. (20) obtain

$$u_{\mathrm{rms}}^2 = r_{m0}^2 - 2r'^T F_A + F_A^T A' F_A \tag{23}$$

where

$$r' = S^T r \tag{24}$$

and

$$A' = S^T A S \tag{25}$$

so that the required control forces, found by minimizing u_{rms} are obtained by solving

$$A' F_A = r' \tag{26}$$

Optimum Placement of Controls

Worst Disturbance

The optimum locations of the heating elements are a strong function of the disturbance which should be designed against. Regardless of the number and placement of the heaters, it is always possible to find a disturbance which renders them ineffective. This can be readily seen from Eq. (12). If the disturbance is orthogonal to all of the function u_j, $j = 1,...,n$, then, $r = 0$ and, therefore, $\Delta T = 0$, and the controls cannot do anything. The same situation exists, of course, for force control. Because of this problem one cannot design the heaters and actuators for the worst disturbance.

The condition that makes the actuators ineffective is the orthogonality of the disturbance to all the functions u_i. Each one of these functions has a peak at the respective control point so that for ψ to be orthogonal to all of them it has to change signs rapidly. Such a rapidly varying function is not likely to have a large amplitude in a real application. In the present paper it is assumed that the disturbance ψ is picked from a class of functions which are limited to have some measure of slowness of spatial variation or a minimum wavelength. One convenient way to apply such a limit to the disturbance is to assume that it is a linear combination of the first few vibration modes of the structure. More generally, it is assumed here that the disturbance ψ is a linear combination of a set of m known functions h_i, $i = 1,...,m$

$$\psi = \sum_{i=1}^{m} h_i \psi_i \tag{27}$$

where ψ_i, $i = 1,...,m$ are some arbitrary coefficients. The design problem which is solved in the following is the selection of the optimum locations of the heaters for the worst possible values of the ψ_i. That is, the best design against the worst disturbance in the class of functions defined by Eq. (27) is sought.

To find the worst disturbance for a given placement of the heaters, the dependence of the rms reduction ratio g on the h_i has to be established. The rms value of the disturbance r_{m0} is

$$r_{m0}^2 = \frac{1}{v_0} \int \psi^2 \mathrm{d}\Omega = \Psi^T H \Psi \tag{28}$$

where Ψ is a vector with components ψ_i, $i = 1,...,m$ and the components h_{ij} of the matrix H are

$$h_{ij} = \frac{1}{v_0} \int h_i \cdot h_j \mathrm{d}\Omega \tag{29}$$

The vector r, defined by Eq. (12), is given in terms of Ψ as

$$r = -K\Psi \tag{30}$$

where the component k_{ij} of the matrix K is

$$k_{ij} = \int_\Omega u_i \cdot h_j \mathrm{d}\Omega \tag{31}$$

The derivation is continued, first for the case of thermal actuators. Using Eq. (16) for g^2, and employing Eqs. (10), (28), and (30) obtains

$$g^2 = 1 - (\Psi^T K^T A^{-1} K \Psi / \Psi^T H \Psi) \tag{32}$$

The worst disturbance vector Ψ maximizes g^2 or, alternatively, can be found from the following formulation.

$$\text{Minimize } \Psi^T K^T A^{-1} K \Psi \text{ such that } \Psi^T H \Psi = 1 \tag{33}$$

This leads to the eigenvalue problem

$$[B - \mu H]\Psi = 0 \tag{34}$$

where $B = K^T A^{-1} K$. The worst disturbance, then, is the eigenvector Ψ, corresponding to the lowest eigenvalue μ_1 of Eq. (34). The rms reduction factor corresponding to Ψ is

$$g = \sqrt{1 - \mu_1} \tag{35}$$

The preceding analysis assumes that each one of the generators of the disturbance space h_i is free of rigid body motion components. Often it is convenient to pick a class of functions that does not satisfy this requirement. Then, the minimization in Eq. (33) has to be augmented with the condition that

$$\int_\Omega \rho\psi \cdot R_i \mathrm{d}\Omega = 0 \qquad i = 1,...,6 \tag{36}$$

or in matrix notation $P\Psi = 0$, where the components p_{ij} of the matrix p are given as

$$p_{ij} = \int_\Omega \rho R_i \cdot h_j \mathrm{d}\Omega \tag{37}$$

The condition $P\Psi = 0$ can be accommodated by employing an additional Lagrange multiplier, but, if a nonsingular submatrix P_B of P is easy to find, then,

$$[P_A P_B] \begin{Bmatrix} \Psi_A \\ \Psi_B \end{Bmatrix} = 0 \tag{38}$$

where Ψ_A and Ψ_B are the corresponding parts of Ψ. Then,

$$\Psi = \begin{bmatrix} I \\ -P_B^{-1} P_A \end{bmatrix} \Psi_A \equiv Q \Psi_A \tag{39}$$

Using this transformation the eigenvalue problem, Eq. (34) is replaced by

$$[B' - \mu H']\Psi_A = 0 \tag{40}$$

where

$$B' = Q^T B Q \qquad H' = Q^T H Q \tag{41}$$

Equations (32-41) were derived for thermal rather than force controls. The changes to accommodate force controls are minimal. The expression for the rms reduction factor g obtained from Eqs. (15) and (23) is

$$g^2 = 1 - (r'^T F_A / r_{m0}^2) \tag{42}$$

Then, using Eqs. (24-26), (28), and (30) the following is obtained.

$$g^2 = 1 - (\Psi^T K'^T A^{-1} K' \Psi / \Psi^T H \Psi) \tag{43}$$

where

$$K' = S^T K \tag{44}$$

All the subsequent equations remain unchanged except that K' replaces K.

Optimization Procedure

The optimization of the location of the controls should have as its goal the minimization of the rms reduction factor g. However, as seen from Eq. (35), this would correspond to the maximization of the lowest eigenvalue μ_I. Such a formulation can become inherently ill-conditioned because the optimization would tend to drive the design to have a multiple lowest eigenvalue. In such cases the derivative of μ_I is not defined (see Ref. 16) and most optimization procedures may encounter some difficulties. To avoid this difficulty the objective function to be minimized was chosen to be

$$f=\sum_{i=1}^{m} g_i^p \tag{45}$$

where g_i is the rms reduction factor corresponding to the ith eigenvector of Eq. (34). As p is increased the objective function is dominated by the largest gain but it also becomes more ill conditioned. The optimization was performed using the NEWSUMT package[17] which is based on an extended penalty function formulation and Newton's method for unconstrained minimization. The only constraints employed are side constraints that keep the location of the controls inside Ω.

Application to Free-Free Beam

Governing Equations

The design procedure is applied to the control of a free-free beam shown in Fig. 1. The beam is assumed to deform only in the x-z plane and the displacement to be controlled is the z displacement of the neutral axis, denoted w. The differential equation that controls the thermal deformation of the beam is

$$\frac{d^2 w}{dx^2} = -\frac{M_{Ty}}{EI_y} \tag{46}$$

where E is Young's modulus, I_y the moment of inertia of the beam's cross section about the y axis, and

$$M_{Ty}=\int_S \alpha_c E_c \Delta T z \mathrm{d}S \tag{47}$$

Here S is the cross-sectional area of the beam, and α_c and E_c the coefficients of expansion and Young's modulus of the control, respectively. It is assumed that the heating is localized to a small volume at each control point. Assuming that at control point i, the heated part of the beam cross section is at coordinates x_i, z_i and has an area ΔS_i and length ΔL_i, then, approximately,

$$M_{Ty_i}=\alpha_c E_c z_i \Delta S_i \Delta T_i \tag{48}$$

This thermal moment produces a slope change

$$\left.\frac{dw}{dx}\right|_{x_i^+} = \left.\frac{dw}{dx}\right|_{x_i^-} - \gamma_i \Delta T_i \tag{49}$$

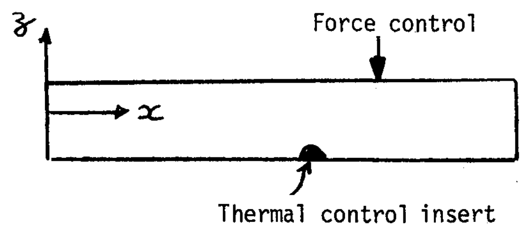

Fig. 1 Beam geometry.

where

$$\gamma_i=\alpha_c E_c z_i \Delta S_i \Delta L_i / EI_y \tag{50}$$

The next step is to find the displacement w_{T0i}, due to a unit ΔT_i with some convenient boundary conditions. Here, it is convenient to assume that the displacement and slope are zero at $x=0$. The solution for a unit ΔT_i is, then,

$$\begin{aligned} w_{Ti0} &= 0 && x \le x_i \\ &= -\gamma_i (x-x_i) && x > x_i \end{aligned} \tag{51}$$

It is assumed that the effect of the change in density due to the control elements is only a minor effect and that the beam has a uniform cross section. Under these assumptions, the condition of rigid body mode orthogonality, Eq. (4) becomes

$$\frac{1}{L}\int_0^L R_i R_j \mathrm{d}x = \delta_{ij} \tag{52}$$

The rigid body modes that affect our problem are the plunge and pitch modes

$$R_1 = 1 \qquad R_2=\sqrt{12}\,[(x/L)-0.5] \tag{53}$$

A concentrated control force F_i at a point i generates a discontinuity in the third derivative. That is

$$\left.\frac{d^3 w}{dx^3}\right|_{x_i^+} = \left.\frac{d^3 w}{dx^3}\right|_{x_i^-} - \frac{F_i}{EI} \tag{54}$$

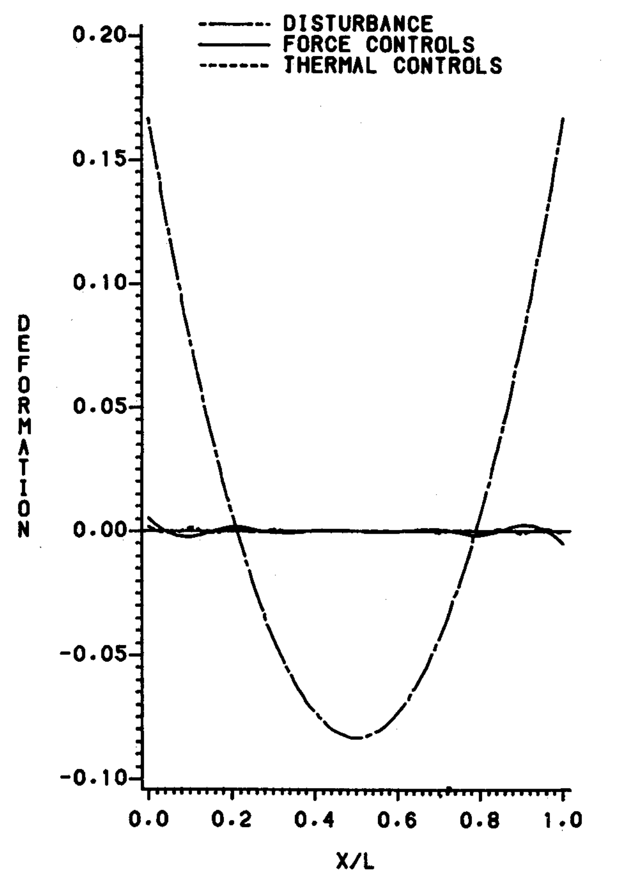

Fig. 2 Control of worst cubic disturbance by nine equidistanced controls.

We again choose the boundary conditions to be clamped at $x=0$, which results in a displacement w_{Fi0} of the form

$$w_{F0} = \frac{1}{EI}\left(\frac{x^3}{6} - \frac{x^2 x_i}{2}\right) \quad \text{for } x \le x_i$$
$$= \frac{x_i^2}{6EI}(x_i - 3x) \quad \text{for } x > x_i \qquad (55)$$

For the beam problem it is convenient to use regular polynomials (that is, $h_i = x^{i-1}$) for the disturbance. The requirement that the disturbance does not have too short a wavelength is accommodated by limiting the order of the polynomials. The expressions for the various matrices used in the analysis are given in the Appendix.

Results

Equidistant Controls

The effect of the shape of the disturbance on the effectiveness of a given set of controls is explored first. It is assumed that a set of equidistant controls is used, such that

$$x_i = iL/(n+1) \qquad i=1,\ldots,n \qquad (56)$$

The worst cubic polynomial disturbance turns out to be the same for any number of force or thermal actuators. The cubic polynomial disturbance and the effects of nine force actuators or nine thermal actuators on it are shown in Fig. 2. The thermal control is seen to be more effective than the force control. The cusps in the deformation pattern with thermal controls are due to the idealization of the thermal actuator at a point of zero width. With the idealization of point controls the thermal actuators produce a discontinuity in the first derivative of the deformation while the force actuators produce a discontinuity only in the third derivative. The rms reduction factor for the worst cubic disturbance is also given in Table 1 as a function of the number of controls. Again, it is clear that thermal controls are more effective.

As the wavelength of the disturbance becomes shorter the effectiveness of the actuators declines rapidly. Table 2 shows the effectiveness of the nine force actuators and of the nine thermal actuators against the worst polynomial disturbance of varying order. It is clear that nine actuators are not sufficient to control the worst sextic disturbances. Unlike the case of the worst cubic disturbance, the shape of the worst disturbance of higher orders does depend on the number of actuators and their type. Figure 3 shows the worst quintic disturbances for nine actuators, and the effect of the actuators on those disturbances. It is seen that the worst disturbances for force and thermal actuators are very similar in shape. The amplitudes of the disturbances are normalized so that the largest monomial coefficient is one. Comparing Figs. 2 and 3 it is seen that with this normalization the worst cubic has a much larger maximum value than the worst quintic. This is due to the fact that the signs of the monomials in the worst polynomial tend to alternate, so that the contribution of the different monomial cancel. This pattern of alternating signs is responsible for producing the shorter wavelength of the higher order worst polynomial.

Table 1 rms reduction factor for worst cubic disturbance $\left(\frac{1}{30} - \frac{2}{5}x + x^2 - \frac{2}{3}x^3\right)$ with equidistanced actuators

Number of controls	g thermal	g force
1	1.00	—
2	0.413	—
3	0.225	1.000
4	0.142	0.423
5	0.0975	0.344
6	0.0711	0.221
7	0.0542	0.173
8	0.0427	0.129
9	0.0345	0.103

Table 2 rms reduction factors for nine equidistanced controls as a function of order of disturbance (worst polynomial)

Polynomial order	g thermal	g force
2	0.0100	0.0204
3	0.0345	0.103
4	0.0855	0.271
5	0.177	0.515
6	0.321	0.752

Table 3 Optimum control locations for worst quintic polynomial disturbance and nine controls

Force control					
x_i/L	0.0174	0.0560	0.186	0.376	0.5
g			0.0461		
Thermal control					
x_i/L	0.0600	0.136	0.267	0.374	0.5
g			0.0875		

Optimally Placed Controls

The optimum location of the controls was calculated for nine control points. Because of the symmetry of the problem, it was assumed that the controls are also symmetric, so that for every control at point x there is also a control at point $1-x$. The exponential p in the objective function was varied from 2 to 4 and, then 6. There was an improvement in the rms

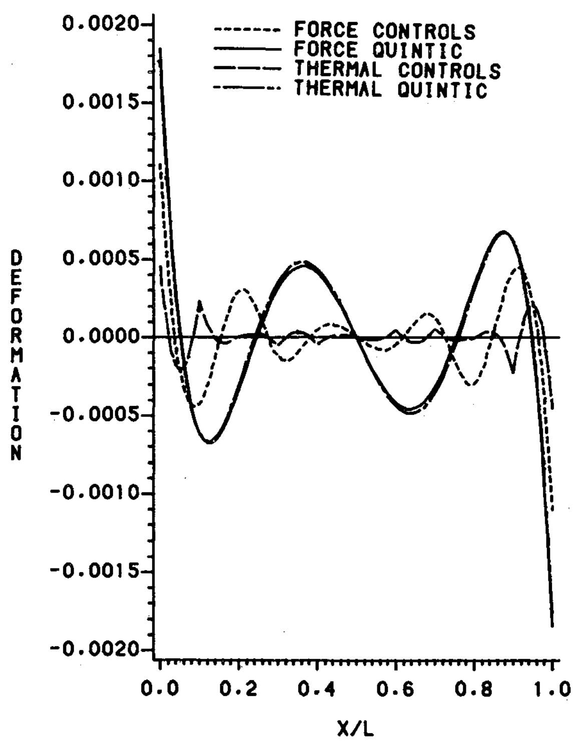

Fig. 3 Control of worst quintic disturbance by nine equidistanced controls.

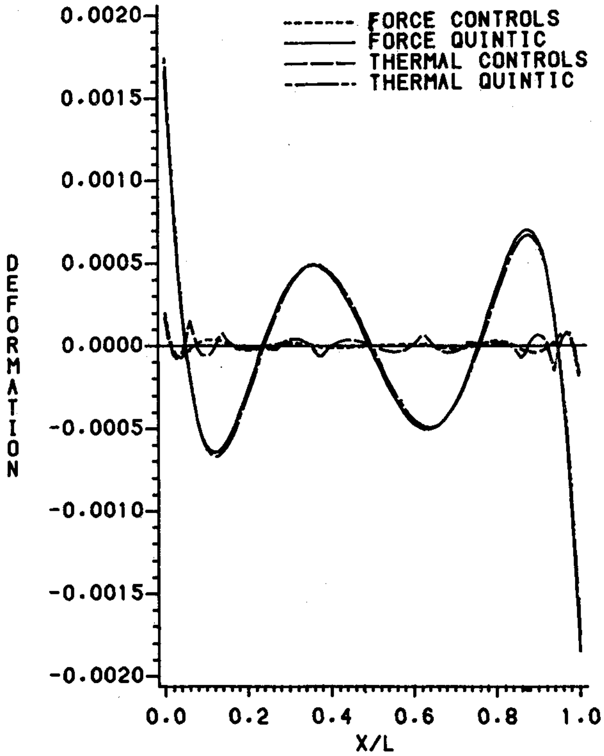

Fig. 4 Optimum 9-actuator control of worst quintic disturbance.

reduction factor going from $p=2$ to $p=4$. However, when p was set to 6 there was only a slight improvement in the rms reduction factor for the thermal controls and a deterioration for the force controls. This poor performance with $p=6$ is due to the ill conditioning of the problem as p gets larger. The results that are presented in the following are for $p=4$, and they are not, therefore, the absolute minimum (which would be obtained by letting p become very large).

The results of the optimization are presented in Table 3. They show the control and the rms reduction factor of nine controls designed against the worst quintic disturbance. The improvement over equidistant controls is particularly noticeable for force controls where the rms reduction factor was reduced from 0.515 to 0.0461. For thermal controls the effectiveness was only doubled, from a reduction factor of 0.177 to 0.0875. The worst quintic disturbances for the optimal control locations and the effects of the controls are also shown in Fig. 4. The results tend to indicate that force controls are more sensitive to locations, and, therefore, the potential for optimization is greater.

Concluding Remarks

A procedure for selecting the optimal locations of actuators for shape control against static disturbances on a space structure was derived. The procedure is based on design against the worst disturbance of a given class of functions and is applicable to both force and thermal actuators. A beam example was used to demonstrate the procedure. The results indicate that the effectiveness of a given set of actuators declines rapidly with decreasing wavelength of the worst disturbance. The optimal selection of the location of the controls showed a very large improvement in control effectiveness, especially for force controls.

Appendix: Matrices Used in Beam Example

Matrix A of Eq. (13) is given in Ref. 4. The matrix K as defined by Eq. (31) is

$$k_{ij}=\frac{1}{L}\int_0^L u_i h_j \mathrm{d}x=\frac{1}{L}\int_{x_i}^L -\gamma_i(x-x_i)x^{j-1}\mathrm{d}x$$

$$=\frac{-\gamma_i}{L}\left[\frac{L^{j+1}}{j+1}-\frac{x_iL^j}{j}-\frac{x_i^{j+1}}{j+1}+\frac{x_i^{j+1}}{j}\right] \quad \text{(A1)}$$

$$h_{ij}=\frac{1}{L}\int_0^L x^{i-1}x^{j-1}\mathrm{d}x=\frac{L^{i+j-1}}{i+j-1} \quad \text{(A2)}$$

The matrix P is defined by Eq. (37)

$$P_{ij}=\frac{1}{L}\int_0^L R_i x^{j-1}\mathrm{d}x \quad \text{(A3)}$$

Using Eq. (51) obtain

$$P_{1j}=\frac{1}{L}\int_0^L x^{j-1}\mathrm{d}x=\frac{L^{j-1}}{j} \quad \text{(A4)}$$

$$P_{2j}=\frac{\sqrt{12}}{L}\int_0^L (x/L-0.5)x^{j-1}\mathrm{d}x=\sqrt{12}L^{j-1}\left(\frac{1}{j+1}-\frac{1}{2j}\right) \quad \text{(A5)}$$

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